Talk for Austalasian Applied Statistics Conference:
Statistics, the law and logic

15 July 2011
SWOT analysis

• Strengths

• Weakness

• Opportunities

• Threats
The Amateurs

• Otago Daily Times 1/11/2010

• “A new University of Otago study has highlighted an association between higher numbers of nearby liquor outlets and increasing risk of binge drinking”

• “With each extra off-licence alcohol outlet within 1 km, the odds of binge drinking increases by 4%, the study lead author, Prof. xxxxx, (Dept of Preventative and Social Medicine) said”

• “Citizens should ask new councils to act now, by developing local alcohol plans that limit the number and location of outlets”
Climatology

- I am not commenting about the conclusions! Just the misuse of statistics
Births per 1000 unmarried NZ women since WW2

Kg bananas consumed per head per year in NZ
What I plan to talk about

• Statistics and the law

• Statistics and logic
Statistics and the Law. Paua Poachers
Paua Poachers

230 Kg Paua Meat - Sold by Agent to Blackmarket Buyer
Paua Poachers
Paua Poachers

Boxplot for Quantity

- Commercial
- Dealer
- Poacher
- Recreational
Percentage of Maximum Penalty vs Quantity of Paua Dealers
Percentage of Maximum Penalty vs Quantity of Paua Poachers
Percentage of Maximum Penalty vs Quantity of Paua Recreational
Something was missing

• Forfeitures were not always recorded and if they were the value was not entered. eg boats, cars, scuba gear etc..

• Suspect the major missing variable for modelling the data was the health and temper of the judge on the day

• The appellate judge presiding over the appeal to which we presented the findings of our study was outraged that we should try to mathematically model his exquisitely considered judgements. He reminded us sternly that “there is far more to sentencing than applying a mathematical model”
Statistics and the Law. Paint Samples

- 49 records of analysis of 27 elements from blue paint samples obtained from under the scales of confiscated fish and the two vessels of the same colour.

- The question was from which boat had the fish been caught and dragged over the stern thus imbedding paint under their scales.

- The presence of so many elements is because of the fillers used in the anti fouling paint on the boats hulls.

- An obvious application for discriminant analysis
FIGURE 1. Barium by Sample Source

FISH

GARRAWAY

KAITI
FIGURE 2. Strontium by Sample Source

- Fish
- Garraway
- Kaiti

Values: 12, 18
FIGURE 3. Galium by Sample Source
FIGURE 4. Indium by Sample Source

[Box plots showing indium levels by sample source: Fish, Garraway, and Kaiti.]

- **Fish**: Indium levels range from 0.0001 to 0.0004.
- **Garraway**: Indium levels range from 0.0002 to 0.0005.
- **Kaiti**: Indium levels are significantly higher, ranging from 0.0001 to 0.0006.
FIGURE 5. Tin by Sample Source

- Fish: 0.035
- Garraway: 0.040
- Kaiti: 0.020
Commentary

• My expectation was that the outcome of this case would have been known long before I was to talk about it.

• It is now due to be heard in September
Statistics and the Law. Operation Webb
Operation Webb
Operation Webb

• Over a 10 year period Observers were sometimes on board. They recorded:
  – Start and finishing date of voyage
  – FMA where each tow was performed
  – Target species
  – Statutory area within each FMA for each tow
  – The quantity of each species caught and processed from each tow

• For example between voyage start date and end date in a particular year the following catch effort was claimed

<table>
<thead>
<tr>
<th>Statutory Area</th>
<th>028 (FMA 5)</th>
<th>030 (FMA 5)</th>
<th>602 (FMA 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td># Tows</td>
<td>44</td>
<td>12</td>
<td>32</td>
</tr>
</tbody>
</table>
Operation Webb

• For unobserved voyages we knew:
  – Start and finishing date of voyage
  – FMA where each tow was claimed to be performed
  – Claimed target species
  – Statutory area within each FMA where each tow was claimed to be made
  – The claimed quantity of each species caught and processed from each tow

• We were keen to judge if the claimed catch when no observer was on board was from the FMA for which it was attributed
Operation Webb

• We could simulate each unobserved voyage by sampling the observer tow data between the dates of the start and end of each unobserved voyage.

• We had observer data for ten years.

• For example on a particular voyage involving 56 tows it was claimed that the total weight of LING caught in FMA 5 was 15 tonnes.

• From 1000 simulated voyages each of 56 tows in FMA 5 we “caught”

<table>
<thead>
<tr>
<th>Species</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ling</td>
<td>123 tonnes</td>
<td>205 tonnes</td>
<td>285 tonnes</td>
</tr>
</tbody>
</table>
# Operation Webb

<table>
<thead>
<tr>
<th>Claimed for FMA 5</th>
<th>Simulated Min</th>
<th>Simulated Mean</th>
<th>Simulated Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 t</td>
<td>123 t</td>
<td>205 t</td>
<td>285 t</td>
</tr>
<tr>
<td>43 t</td>
<td>36 t</td>
<td>82 t</td>
<td>170 t</td>
</tr>
<tr>
<td>28 t</td>
<td>112 t</td>
<td>188 t</td>
<td>274 t</td>
</tr>
<tr>
<td>25 t</td>
<td>99 t</td>
<td>188 t</td>
<td>288 t</td>
</tr>
<tr>
<td>15 t</td>
<td>35 t</td>
<td>73 t</td>
<td>126 t</td>
</tr>
<tr>
<td>29 t</td>
<td>10 t</td>
<td>24 t</td>
<td>39 t</td>
</tr>
<tr>
<td>154 t</td>
<td>69 t</td>
<td>115 t</td>
<td>165 t</td>
</tr>
<tr>
<td>100 t</td>
<td>98 t</td>
<td>158 t</td>
<td>241 t</td>
</tr>
<tr>
<td>43 t</td>
<td>60 t</td>
<td>130 t</td>
<td>207 t</td>
</tr>
</tbody>
</table>
## Operation Webb

<table>
<thead>
<tr>
<th>Claimed for FMA 6</th>
<th>Simulated Min</th>
<th>Simulated Mean</th>
<th>Simulated Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>492 t</td>
<td>53 t</td>
<td>111 t</td>
<td>196 t</td>
</tr>
<tr>
<td>195 t</td>
<td>37 t</td>
<td>81 t</td>
<td>153 t</td>
</tr>
<tr>
<td>326 t</td>
<td>52 t</td>
<td>114 t</td>
<td>183 t</td>
</tr>
<tr>
<td>339 t</td>
<td>97 t</td>
<td>164 t</td>
<td>246 t</td>
</tr>
<tr>
<td>173 t</td>
<td>24 t</td>
<td>56 t</td>
<td>96 t</td>
</tr>
<tr>
<td>85 t</td>
<td>17 t</td>
<td>34 t</td>
<td>52 t</td>
</tr>
<tr>
<td>94 t</td>
<td>8 t</td>
<td>25 t</td>
<td>50 t</td>
</tr>
<tr>
<td>513 t</td>
<td>105 t</td>
<td>168 t</td>
<td>236 t</td>
</tr>
<tr>
<td>419 t</td>
<td>59 t</td>
<td>100 t</td>
<td>155 t</td>
</tr>
</tbody>
</table>
## Operation Webb

<table>
<thead>
<tr>
<th>Claimed for Voyage</th>
<th>Simulated Min</th>
<th>Simulated Mean</th>
<th>Simulated Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>507 t</td>
<td>215 t</td>
<td>314 t</td>
<td>434 t</td>
</tr>
<tr>
<td>238 t</td>
<td>97 t</td>
<td>163 t</td>
<td>277 t</td>
</tr>
<tr>
<td>353 t</td>
<td>208 t</td>
<td>301 t</td>
<td>431 t</td>
</tr>
<tr>
<td>364 t</td>
<td>254 t</td>
<td>364 t</td>
<td>504 t</td>
</tr>
<tr>
<td>188 t</td>
<td>72 t</td>
<td>129 t</td>
<td>196 t</td>
</tr>
<tr>
<td>114 t</td>
<td>39 t</td>
<td>58 t</td>
<td>83 t</td>
</tr>
<tr>
<td>248 t</td>
<td>89 t</td>
<td>140 t</td>
<td>193 t</td>
</tr>
<tr>
<td>613 t</td>
<td>219 t</td>
<td>326 t</td>
<td>401 t</td>
</tr>
<tr>
<td>461 t</td>
<td>139 t</td>
<td>229 t</td>
<td>336 t</td>
</tr>
</tbody>
</table>
Operation Webb

• We were keen to argue in court that a foul had been committed
• Previously we had been told that such Monte Carlo studies were “not evidential” and they were “novel science” but we wanted to at least put our case
• The evidence was not intended to stand on its own
  – The GPS position when Electricity consumption for Blast Freezers was high was particularly incriminating
  – Conflicting catch records...one official and one for Japanese principals consumption

• Before the anticipated trial a guilty plea was made. The company agreed to pleaded guilty to “trucking” 500 tonnes of Ling.

• Penalties imposed included forfeiture of the trawler and catch.
  – Totalled more than $4,200,000

• The issue for us is once again how to penetrate the statistics/law barrier.
  – No irritants are allowed through...it behaves like the blood/brain barrier
Statistics and the Law. Maxicrop Trial 1987

- Dr. Doug Edmeades who worked for the Ministry of Agriculture and Fisheries performed 18 trials in which there was a no significant response
  - It was unlikely that the trials had power to detect a 20% response

- Bell Booth who were the manufacturers of Maxicrop performed 1 trial in which there was a 10% response...and no standard errors

- As a result Dr. Edmeades had this to say on “Fair Go”
Maxicrop Trial 1987

• “Maxicrop contains low concentrations of plant nutrients and plant growth regulators”

• “When applied at recommended rates, Maxicrop has no effect on pasture production, chemical composition and plant nutrient uptake”

• “Based on this evidence, MAF would not recommend the use of Maxicrop in pastoral agriculture”
Maxicrop Trial 1987

- Maxicrop's manufacturer unsuccessfully sued MAF and Television New Zealand for $11.5 million over claims made on “FairGo”

- The court case cost millions of dollars, took 135 days and is one of New Zealand's longest-running cases

- The High Court found that Dr Edmeades' statements were true and that Maxicrop did not work

- So the statisticians who appeared for the defence convinced a judge to accept $H_0$
Balance of probabilities
Statistics and the Law

• Irrational fear of mathematical models
  – Like the dead ferrets they pull over their head reviving and running away

• I hope they can understand some simple statistics

• The idea of Monte Carlo simulation is abhorrent.

• Statisticians and the law have a different understanding of probability

• What should be done about it?
Statistics and the Law

• Working Group on Statistics and the Law set up by RSS in 2004 to address some of the issues

• First of 4 reports dated November 2010


  – Colin Aitken Professor of Forensic Statistics, U of Edinburgh. Chairman
  – Paul Roberts Professor of Criminal Jurisprudence. U of Nottingham
  – Graham Jackson Professor of Forensic Science, Abertay University

• Is designed as a general introduction to the role of probability and statistics in criminal proceedings.
Statistics and the Law

- Guide 2: DNA profiling evidence
- Guide 3: Networks for structuring evidence
- Guide 4: Case assessment and interpretation
Statistics and Logic


• “For the logical fallacy of believing that a hypothesis has proved to be true, merely because it is not contradicted by the available facts, has no more right to insinuate itself in statistical than in other kinds of scientific reasoning”

• Absence of evidence is not evidence of absence of an effect!
Statistics and Logic

- The Logic of Scientific Discovery, Karl Popper, 1934
  - English translation 1959

- Laws can be conclusively falsified but cannot be conclusively verified. Although laws give good predictions they may one day fail to do so
Statistics and Logic

• Both gurus affirm that although we can reject $H_0$ we cannot accept $H_0$

• We all know in many cases it is ridiculous to think the $H_0$ is true
Statistics and Logic

• Why then do we accept $H_0$ when we apply Tests of Fit for a distribution?
  – Pearson’s Chi-squared Test
  – Fisher’s Exact Test
  – Kolmogorov-Smirnoff Test
  – Cramer-von-Mises Criterion
  – Anderson-Darling Test
  – Mann-Whitney U
Statistics and Logic

• My first boss showed me how to fiddle Pearson’s $\chi^2$ test of fit to get the result one hoped for

• Generate 100 random values from $\chi^2_5$
Histogram of 100 samples from a Chi Square 5 distribution
Pearson’s test of fit

• Calculated the sample mean and sample variance

• 4 classes with equal probabilities from a normal distribution
• $\chi^2$ test of fit criterion of 8.00 on 1 df
  – So I am able to emphatically reject $H_0$ the fit is good.

• 8 classes with equal probabilities from a normal distribution
• $\chi^2$ test of fit criterion 8.16 on 5 df
  – So I am unable to reject $H_0$
Statistics and Logic

• Why then do we accept $H_0$ when we apply Tests where $H_0$ is variances are equal?
  – Bartlett’s Test.
  – Levene’s Test
  – Brown-Forsythe Test

• What is wrong with Monte Carlo studies if there is any doubt?
Statistics and Logic

• Why then do we accept $H_0$ when we apply Tests where $H_0$ is a random series?
Statistics and Logic

- Questions of power of the test in any particular situation are unknown
- The consequences of accepting $H_0$ when $H_0$ is untrue, are unknown
- There is an infinite number of ways of deviating from $H_0$

- When I first started as a statistician I was confused by the conflicting behaviour;
  - some times it was unacceptable to accept $H_0$
  - some times it was unacceptable to not accept $H_0$
- It took me years to resolve the conflict
- How can we expect casual users of statistics to understand?
Statistics and Logic

• We should all continue to discourage the fallacious interpretation of significance test

• Perhaps a good way to start is to consign tests of fit to a respectable museum
Didymosphenia

- What prevalence of didymosphenia would be consistent with the algae not having been detected in a sample of 64 rivers?
  - Assume sample design and strategy has been adequate
• Assume a Beta (a;b) prior.
  – If a = b = 1 we have a uniform prior.
  – If a = b = ½ we have the Jeffrey’s prior

• n = total number of sites tested and n₀ is the number of sites where Didymo detected

• With a Beta prior and a binomial likelihood the posterior is Beta (n₀+a; n-n₀+b).

• Suppose n = 64 and n₀ = 0
## Statistics and Logic

<table>
<thead>
<tr>
<th>Prior Type</th>
<th>Value</th>
<th>Upper 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeffrey’s prior</td>
<td>0.0000</td>
<td>0.02945</td>
</tr>
<tr>
<td>Uniform prior</td>
<td>0.0000</td>
<td>0.04504</td>
</tr>
</tbody>
</table>
Statistics and Logic

• Would the recipients of a statistical education benefit from a concurrent course in Scientific Logic?